QUINE on MATHEMATICAL RECREATION

from REPLY to CHARLES PARSONS
in HAHN + SCHILPP 1986

I have said that I am belittling the difference between mathematics and natural science; but not denying it. And I am belittling it only to some degree. At the end of *Philosophy of Logic* I contrasted mathematics and logic with the rest of science on the score of their versatility: their vocabulary pervades all branches of science, and consequently their truths and techniques are consequential in all branches of science. This is what has led people to emphasize the boundary that marks pure logic and mathematics off from the rest of science. This also is why we are disinclined to tamper with logic or mathematics when a failure of prediction shows there is something wrong with our system of the world. We prefer to seek an adequate revision of some more secluded corner of science, where the change would not reverberate so widely through the system.

This is how I explain what Parsons points to as the inaccessibility of mathematical truth to experiment, and it is how I explain its aura of a priori necessity. In what sense, after all, is a statement even of theoretical physics accessible to experiment? Let S be such a statement. The physicist proceeds to test it, as he says, by arranging certain observable conditions and then seeing whether a predicted observation ensues. But the prediction is not a logical consequence simply of S and the arranged experimental conditions. A substantial body of associated physical and mathematical premisses is needed, not just S, in order to clinch the implication. Failure of the prediction will show only that this substantial body of physical and mathematical theory is not tenable intact. What is special about the particular statement S is just that the physicist has chosen to finger it. Because perhaps of considerations of simplicity or symmetry or analogy or confinement of reverberations, S happens to be the one component statement of his theory that the physicist would be happiest to revoke if the experiment shows that the theory is not tenable intact. And I have said why he may be counted on not to choose S from the purely mathematical part of his inclusive theory.

Pure mathematics, in my view, is firmly imbedded as an integral part of our system of the world. Thus my view of pure mathematics is oriented strictly to application in empirical science. Parsons has remarked, against this attitude, that pure mathematics extravagantly exceeds the needs of application. It does indeed, but I see these excesses as a simplistic matter of rounding out. We have a modest example of the process already in the irrational numbers: no measurement could be too accurate to be accommodated by a rational number, but we admit the extras to simplify our computations and generalizations. Higher set theory is more of the same. I recognize indenumerable infinites only because they are forced on me by the simplest known systematizations of more welcome matters. Magnitudes in excess of such demands, e.g., $\beth_\omega$ or inaccessible numbers, I look upon only as mathematical recreation and without ontological rights. Sets that are compatible with 'V = L' in the sense of Gödel's monograph afford a convenient cut-off.